An AHP-based decision-making tool for the solution of multiproduct batch plant design problem under imprecise demand

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Abstract

This paper addresses the problem of the optimal design of batch plants with imprecise demands in product amounts. The design of such plants necessarily involves the way that equipment may be utilized, which means that plant scheduling and production must form an integral part of the design problem. This work relies on a previous study, which proposed an alternative treatment of the imprecision (demands) by introducing fuzzy concepts, embedded in a multi-objective Genetic Algorithm (GA) that takes into account simultaneously maximization of the net present value (NPV) and two other performance criteria, i.e. the production delay/advance and a flexibility criterion. The results showed that an additional interpretation step might be necessary to help the managers choosing among the non-dominated solutions provided by the GA. The analytic hierarchy process (AHP) is a strategy commonly used in Operations Research for the solution of this kind of multicriteria decision problems, allowing the apprehension of manager subjective judgments. The major aim of this study is thus to propose a software integrating the AHP theory for the analysis of the GA Pareto-optimal solutions, as an alternative decision-support tool for the batch plant design problem solution.

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1. Introduction

While quantitative consideration of uncertainty in the design of continuous processes has received increased attention in recent years [1], considerably less attention has been granted to this feature in the context of batch processing. Yet, this issue is of particular relevance for contemporary multiproduct batch operations due to the high degree of uncertainty arising from the complexity and typically incomplete information about the involved chemical/physical steps, the heavy reliance on timely and appropriate operator initiated actions, the need to process multiple products in the same facility as well as the relative volatility of the demands for the specialized products typically produced according to the batch mode [2].

The growth of specialty chemicals, food products and pharmaceutical industries aroused the current focus on the batch plant design problem. In the conventional optimal design of a multiproduct batch chemical plant, a designer...
specifies the production requirements for each product and total production time for all products. The number, required volume and size of parallel equipment units in each stage are to be determined in order to minimize the investment cost. Such an approach formulates the optimal design problem as a single-objective mixed-integer nonlinear programming (MINLP) problem [3–10].

However, in real world applications, the chemical engineers often need to make decisions when faced with competing objectives [11]. For instance, the optimal design of a multiproduct batch chemical plant not only accounts for the investment cost minimization, but also for operation cost minimization, makespan minimization and/or revenue maximization. All those objectives might be considered simultaneously [10].

In this framework, this study introduces a new design approach to maximize the net present value (NPV) and two other performance criteria, i.e. the production delay/advance with respect to a fixed date and a flexibility criterion. This design problem becomes a multi-objective optimization problem (MOOP). Multi-objective optimization is a natural extension of the traditional optimization of a single-objective function. If the multi-objective functions are commensurate, combining all the objectives within a single-objective function enables the use traditional optimization techniques. However, if the objective functions are incommensurate, or competing, then an accurate weight factor tuning is necessary to get a good scaling of the criteria involved in the optimized single-objective function.

Besides, the competition between several objectives causes lack of complete order for MOOPs. For instance, in an optimal design problem, the simultaneously required minimization of the investment cost and maximization of the revenue, can lead to antagonist configurations. Pareto optimality or non-inferiority is therefore used to define an optimal solution to MOOPs [12].

On the other hand, the key point in the optimal design of batch plants under imprecision concerns the modeling of demand variations. The market demand for products resulting from the batch industry is usually changeable, and at the stage of conceptual design of a batch plant, it is almost impossible to obtain the precise information on the future product demand over the plant lifetime. Nevertheless, decisions must be taken on the plant capacity. This capacity should be able to balance the product demand satisfaction and extra-capacity in order to reduce the loss on the excessive investment or than on market share due to the varying product demands [13].

The most common approaches treated in the dedicated literature represent the demand uncertainty with a probabilistic frame by means of Gaussian distributions. Yet, this assumption does not seem to be always a reliable representation of the reality, since in practice the parameters are interdependent and do not follow symmetric distribution rules, which leads to very complex conditional probabilities computations.

An alternative treatment of the imprecision is constituted by using fuzzy concepts [14]. This approach, based on the arithmetic operations on fuzzy numbers, differs mainly from the probabilistic models insofar as distribution laws are not used. It considers the imprecise nature of the information, thus quantifying the imprecision by means of fuzzy sets that represent the “more or less possible values”. A previous work [15] proposed the integrated use of the above-mentioned fuzzy concepts into a Genetic Algorithm (GA) for the treatment of multi-objective batch plant design problems, since this stochastic optimization method is particularly well suited to tackle imprecise, multi-objective applications.

But the huge number of Pareto-optimal solutions emerging from the GA naturally leads to the question of choosing the best configuration, i.e. the most adapted to any manager’s wishes. As a result, multicriteria decision-making techniques should be employed in solving this issue. A common approach is the analytic hierarchy process (AHP [16–18]). For this purpose, the results generated by the multi-objective, fuzzy GA are treated in this study by the conventional AHP, which shows the effectiveness and some unique advantages.

The use of AHP as an additional decision-support tool, for different kinds of problems, was highlighted in previous studies: for instance, its application to criteria evaluating postal service efficiency was carried out by Chan [19]. Furthermore, the combination of fuzzy logic and AHP concepts has already proved to be consistent for the treatment of multicriteria problems. The Fuzzy Extended Analytic Hierarchy Process (FEAHP) was proposed on problems of critical decision identification, including risk factors, for the development of an efficient system for global supplier selection [20] (the criteria being quality, service performance and supplier’s profile).

The paper is organized as follows. Section 2 is devoted to a brief description of a study antecedent, the integration of fuzzy set theory within a multi-objective Genetic Algorithm (MOGA). Section 3 presents an overview of the AHP while Section 4 details its application to the treated batch plant design problem. This application is then illustrated by some typical results in Section 5. Finally, the conclusions on this work are drawn.
2. Study antecedent

2.1. Fuzzy logics

The emergence of electronic commerce and business-to-business applications has, in a recent period, considerably changed the dynamics of the supplier–customer relationship. Indeed, customers can change more rapidly their orders to the suppliers and many enterprises have to organize their production even if the demand is not completely known at short term. On the other hand, the increasing need for integration and optimization in supply chains leads to a greater sensitivity to perturbations due to this uncertainty. These two elements clearly show the interest of taking into account as soon as possible the uncertainty on the demand and to propagate it along the production management mechanisms.

In the context of engineering design, an imprecise variable is a variable that may potentially assume any value within a possible range because the designer does not know a priori the final value that will emerge from the design process. The fuzzy set theory was introduced by Zadeh [14] to deal with problems in which a source of vagueness is involved. It is well recognized that fuzzy set theory offers a relevant framework to model imprecision.

In this section, only the key concepts from the theory of fuzzy sets that will be used for batch plant design are presented; more detail can be found in [21]. Different forms can be used to model the membership functions of fuzzy numbers. We have chosen to use normalized trapezoidal fuzzy numbers (TrFNs) for modeling product demand, which can be represented by a membership function \( \mu(x) \).

The proposed approach involves arithmetic operations on fuzzy numbers and quantifies the imprecision of the demand by means of trapezoidal fuzzy sets (see Fig. 1). We represent subjective judgments on future demand, given as linguistic values, such as “demand is around a certain value or interval \([q_2, q_3]\)” or “demand is not lower than a certain value”.

For the design of the demand, we suppose that the products have a sure level of acceptance in market, represented by the interval \([q_2, q_3]\): this means that the demand has, in this interval, a certainty level \( z = 1 \) that derives in TrFNs. On the other hand, the intervals \([q_1, q_2]\) and \([q_3, q_4]\) represent the demand “more or less possible values”. The consequence of representing the demand with a TrFN in the model is to obtain a \( NPV \) with a certainty interval equal to 1, as illustrated in Fig. 1.

The arithmetic calculations involve addition, subtraction and symmetric (image), through the extension principle of Zadeh [14].

- Addition: \( A(+B) = (a_1, a_2, a_3, a_4)(+)(b_1, b_2, b_3, b_4) = (a_1 + b_1, a_2 + b_2, a_3 + b_3, a_4 + b_4) \);
- Subtraction: \( A(−B) = (a_1, a_2, a_3, a_4)(−)(b_1, b_2, b_3, b_4) = (a_1 − b_4, a_2 − b_3, a_3 − b_2, a_4 − b_1) \) - symmetric (image) of a TrFN: \( −(A) = (−a_4, −a_3, −a_2, a_1) \).

2.2. Multi-objective batch plant design under uncertainty

The model used in this study is derived and adapted from that proposed by Modi and Karimi [5]. Actually, this model has its early roots in a work of Grossmann and Sargent [3], who developed a simple formulation for multiproduct batch plant design. This same formulation was adopted and extended by several authors such as Modi and Karimi, who, in addition to the typical batch items, considered semi-continuous stages and storage tanks (with fixed location in the process) to the initial formalism. In a majority of these mono-objective formulations, the typical criterion consists of the minimization of the investment cost corresponding to all items. Furthermore, the solution is subject to a major
constraint, forcing the synthesis time of all products within a time horizon $H$. The decision variables, which define the plant configuration, are the (continuous) size and (discrete) number of items for each processing stage. Since the formulation involves nonlinear functions, the problem typically results in a complex, non-convex MINLP problem.

The model used in this paper, although based on the above-mentioned formulation, was modified according to the proposal of Aguilar-Lasserre et al.\cite{Aguilar-Lasserre}\cite{15}. The production time constraint was removed and integrated in two additional objective functions, detailed next. This strategy enables to handle the constraint with more flexibility and to provide at the end of the search a range of solutions (i.e. non-dominated Pareto solutions) that might not be all feasible but acceptable in an operative viewpoint. A manager should then be able to choose, among the proposed configurations, the more profitable one. Moreover, the used formulation considers uncertainty on the product demand by way of fuzzy logics concepts: the demand is then represented by a trapezoid. The fuzzy nature of this parameter reflects itself in the other variables and on the economic criterion through fuzzy arithmetic, implemented in the model computation.

Actually, three different criteria are considered in the study:

(a) Instead of the investment cost recommended by Modi and Karimi\cite{5}, the economic criterion represents the NPV. This approach allows evaluating the impact of the plant over some years, taking into account the calculated net cash flows in terms of the present value of the money.

$$\text{Max } NPV = -Inv - f + \sum_{p=1}^{n} \frac{(V_p - D_p - A_p)(1 - a) + A_p}{(1 + i)^n} + \frac{f}{(1 + i)^n},$$

Eq. (1) underlines the fact that the objective function accounts not only for the investment cost, but also for the incomes from the sells ($V_p$), the operation costs ($D_p$) and depreciation ($A_p$) computed on $n$ given time periods. Discount rates ($r$), taxes ($a$) and working capital ($f$) are also involved to update the money value. It is worth noting that, since sells and operation costs depend on the uncertain demand parameter, these quantities turn out to be fuzzy numbers. The arithmetical rules thus make the whole NPV criterion to be represented by a TrFN.

(b) The second criterion minimizes the delay or advance of the production time for all products $\sum H_i$, with respect to a fixed horizon time $H$. Being $\sum H_i$ a (trapezoidal) fuzzy quantity, $H$ is transformed into a rectangle such as illustrated in Fig. 2: the criterion can then be converted into the maximization of the common surface between the two fuzzy quantities. The result is multiplied or divided by factor $\omega$ ($\omega > 1$) in the advance or delay case respectively, in order to penalize plant configurations characterized by a delay. Moreover, to ensure the superiority of just-in-time solutions, the criterion is assigned a huge value ($M$) when the trapezoid ($\sum H_i$) is included in the rectangle ($H$). Fig. 3 sums up the various considered cases (just-in-time is case 1). The exact formulation is given in the following equations:

$$\text{Max(Delay crit.)} = \text{common surface} \times \frac{1}{\omega},$$  

$$\text{Max(Advance crit.)} = \text{common surface} \times \omega.$$  

(c) The third criterion represents a flexibility index and measures the capacity of the plant to manufacture an additional production. On the one hand, in advance cases, the remaining time to complete to the time horizon $H$ is available time that can be used to produce $Q_{\text{additional}}$. On the other hand, in delay cases, the production time exceeding $H$ can be interpreted as a demand that could not be fulfilled, $Q_{\text{unsatisfied}}$. The flexibility index is formulated in
Eq. (4), underlining that the flexibility will be greater (resp. lower) than 1 for advance (resp. delay) cases, and equal to 1 for just-in-time cases:

$$\text{flexibility index} = \frac{\sum_{i=1}^{I} (Q_i + Q_i^*)}{\sum_{i=1}^{I} Q_i}.$$  (4)

The complete model is given in Appendix A, with a nomenclature explaining in detail the implementation steps. It is to note that the optimization variables are almost the same with respect to the initial Modi and Karimi formulation [5]:

- Volume $V_j$ of the items of each batch stage $j$ and treatment capacity $R_k$ of each semi-continuous stage $k$. However, these variables are not continuous anymore and were discretized with an interval of 50 units between two possible values. This working mode was adopted in a view of realism. Indeed, since equipment manufacturers propose the items following defined size ranges, the design of operation unit equipments does not require a level of accuracy such as real number. Note however that the initial bounds on these size variables were kept unchanged, being for batch and semi-continuous, respectively: $V_{\text{min}}$ and $V_{\text{max}}$, and $R_{\text{min}}$ and $R_{\text{max}}$.
- Item number $m_j$ in batch stage $j$ and item number $n_k$ in semi-continuous stage $k$. These variables cannot exceed 3 items per stage ($1 \leq m_j, n_k \leq 3$).

The new multi-objective problem is also a non-convex MINLP problem; the optimization technique used for its treatment is described in the next section.

2.3. Problem-specific GA

The choice of a metaheuristic method, and particularly a GA, is justified, first, because the great quality of this stochastic method was already proved for the solution of many batch plant design instances. Furthermore, the framework of the study involves handling of both multi-objective optimization and uncertainty (through fuzzy logics) concepts. Concerning the former, GAs have shown, in many studies, the advantage of easily handling various criteria thanks to their ability to directly provide, at the end of the search, a Pareto solutions set. Then, integrating the fuzzy logics concepts does not imply that much changes with respect to the classical metaheuristic operating mode (only the creation of the specific fuzzy arithmetic operators). Further studies showed that the handling of fuzzy logics concepts is much more difficult to implement in, for instance, a Mathematical Programming framework [22].
General principles on evolutionary computation will not be recalled here, since the aim is only to insist on the particular details of the used GA. Actually, this one is a standard multi-objective algorithm, similar to the specific one developed in [15].

It is characterized by the random generation of the initial population. Since the production time disappeared from the formulation, the only restriction to respect is the item size ranges. However, in order to avoid beginning the search procedure with inappropriate solutions, individuals highly exceeding the time horizon time were rejected for the first population creation.

The encoding procedure is adapted to the variable nature: the item size variables are coded according to a binary-like technique (i.e. the weighted-box method [21]) while the item number per stage are copied just as they are worth in the chromosome (for instance, if \( m_j = 2 \), the corresponding locus will contain information “2”). The resulting configuration of a chromosome is shown in Fig. 4.

The encoding procedure is adapted to the double nature of the variables: since continuous and integer variables have to coexist in the same chromosome, this latter is partitioned into two zones. As shown in Fig. 4, the first zone encodes the continuous variables, i.e. the item sizes of each processing stage, as reduced variables (between 0 and 1, using the lower and upper bounds) and according to a binary-like technique that is not detailed here (the weighted-box method [23]). On the other hand, the integer variables, representing the item number for each stage, are copied directly in the chromosome without any change: for instance, the plant illustrated in Fig. 4 has 2 items for stage 1, 1 item for stage 2, and 3 items for stage 3: this corresponds to the integer numbers encoded at the end of the chromosome: 2, 1, 3.

Besides, all the criteria have to be maximized, so the fitness is equal to the objective function it is associated to.

Concerning genetic operators, the crossover obeys to a classical one cut-point method, while the mutation technique is in agreement with the encoding procedure: inversion of a bit value on the continuous zone (0 becomes 1 and conversely), and decrease of one unit for the discrete loci (if possible, of course).

But, the selection procedure is the only problem-specific procedure, which was developed regarding the multi-objective nature of the problem and considering that a preference order can be established with respect to the possible cases for the second criterion (see Fig. 3).

A classical way of handling multicriteria problems is the domination concept, developed by Pareto [24]: similarly with the MOGA proposed by Fonseca and Flemming [25], the selection of the best individuals in the current population can be done through a Pareto sorting procedure. However, it cannot be ensured that the number of non-dominated solutions will be equal to the survivor number, which is one of the GA parameters. Two successive Pareto rounds are thus implemented, but it is not ensured that the survivor number is reached. To overcome this issue, the delay/advance cases can constitute a solution.
Actually, looking at Fig. 3, it clearly appears that case 1 is more favorable than case 2, that case 2 is more favorable than case 3, etc. This observation can be implemented in a specific tournament, whose rules will select preferably case 1-individuals, then case 2-individuals, . . . , until case 8-individuals.

Finally, the resulting hybrid selection step combines two Pareto sorting stages and one case-based tournament, explained as follows. As depicted in Fig. 5, a first Pareto sorting procedure is carried out on the whole population. Three cases arise from this step: if the number of non-dominated individuals is equal to the survivor number, then the selection step ends. On the other hand, if the non-dominated individual number is higher than the survivor numbers, these non-dominated individuals are ranked according to the cases illustrated in Fig. 3, from case 1 to case 8. The first ones are chosen to complete the survivor numbers. Otherwise (non-dominated individuals number lower than survivor number), all the non-dominated individuals survive and a second Pareto sort is realized to complete the survivor number.

This leads, once again, to three possible cases:

- if the sum of the non-dominated individuals from both Pareto sorts is equal to the survivor number, then the selection step ends;
if the sum of non-dominated individuals from both Pareto sorts is greater than the survivor number, the non-dominated individuals from the second Pareto sort are ranked according to the cases of Fig. 3 and the first ones are chosen to have the survivor set complete;

else (the sum of non-dominated individuals is lower than the survivor number), both non-dominated individual sets are selected to pass to the next generation, and the survivor number is completed with solutions of the second dominated individual set. These solutions are chosen according to a tournament step, based on the cases depicted in Fig. 3.

This procedure ensures getting a valid number of individuals to survive. Moreover, even if some kind of elitism is introduced by the case-based tournament method, the very open Pareto sorting procedure allows to keep some diversity in the population (and thus to avoid getting trapped in any local optimum, leading to premature convergence).

A standard termination criterion was chosen too, i.e. the search stops when a maximal generation number is reached. Finally, a Pareto sort procedure is carried out at the end of the search, in order to determine all the non-dominated solutions visited during the whole search.

The operating parameters of the used GA were tuned with sensitivity analysis, and are available next:

- population size: 200.
- computed generation number: 400.
- crossover rate: 0.60.
- mutation rate: 0.30.

2.4. Study’ aims

The strategy described above was implemented for the study published in [15]. The fuzzy demand parameter was chosen to be asymmetrical, i.e. the distance from the core bounds to the support bounds are not similar for left-hand side and right-hand side.

As an illustrative example, the plant used as a support is an academic instance designed in Ponsich et al. [26]. However, the used model and the plant size make the chosen example relevant and might allow transposing the obtained results to a realistic industrial process. The plant is composed of six batch stages, eight semi-continuous stages and one intermediate storage tank. So, the example has 14 item size and 14 item number variables. All data are available in [15].

Optimizations were first realized for bicriteria cases (NPV–common surface and NPV–flexibility index), and then considering the three criteria at the same time. For the former computations, the number of non-dominated solutions was equal to 238 (NPV–surface) and 277 (NPV–index). This constitutes a huge number of solutions, and a manager should be supposed to find among them the one that better satisfies him.

When considering the tricriteria computation, the number of Pareto-optimal solutions is still greater: logically, when more criteria are contemplated, the number of non-dominated solutions turns out to be higher. Indeed, for this tricriteria computation, the Pareto sorting procedure emphasizes on 1549 solutions. Choosing an adequate configuration thus constitutes a very complex problem for the manager to solve. Furthermore, Fig. 6 shows that the resulting Pareto region illustrated in a 3D-chart is hardly interpretable.

So, the issue of determining the most preferable solution is a harsh task, and even if some intuitive approach in the bicriteria cases can be proposed [15], the work become almost impossible for three simultaneous criteria. These various remarks lead to the necessity of an additional decision-making tool that may help to choose, among the non-dominated solutions, which is the most adapted to the managers’ will.

However, all the solutions deriving from the Pareto sort are supposed to be “equal”. So, purely objective considerations might not enable to decide what is the “best” configuration. It is then necessary to use a procedure that could involve the subjective viewpoint of each particular manager, i.e. a method that would be able to integrate and rationalize the manager’s needs. Moreover, this must be done in a systematic way so as the method could adapt to each particular case of manager’s wish.

Finally, let us recall that the first criterion NPV is a trapezoidal fuzzy quantity. Consequently, the tackled method should be able to take advantage of the shape of the final result and to highlight each one of its characteristics. In the uncertain environment that is the framework of the study, providing an analysis of these factors would be highly valuable for a manager, who could interpret it according to his profile (for instance, risk-averse or risk-taker).
Therefore, the chosen technique, which seems to satisfy all the above-mentioned requirements, is the AHP. The description of this method is presented in the following section.

3. AHP method overview

The AHP is a systematic analysis technique developed for multicriteria decision [27]. Its operating mode lays on the decomposition and structuring of a complex issue into several levels, rigorous definition of manager priorities, and computation of weights associated to the alternatives. The output of AHP is a ranking indicating the overall preference for each decision alternative.

3.1. Scope of the method

The AHP proposes a methodology to organize the analytical thought, according to three basic principles [28]:

- **The hierarchy construction principle**: The AHP underlying assumption is that complex systems can be better understood through decomposition into essential elements. These elements can be the criteria involved in the considered decision problem, and be hierarchically structured into several levels, according to the relative importance of each element with respect to another one. The highest level represents the main decision objective, while the lowest one is constituted by the different alternatives.

- **The priority setting principle**: Human beings are able to intuitively perceive relationships between two elements, to express a preference of one on the other and to numerically evaluate this preference. This is still true regarding subjective considerations, since the idea is to translate a feeling. However, a fixed priority scale must be implemented in order to make the evaluation independent from the different orders of magnitude that characterize each element. From the synthesis of this pairwise judgment set is derived the priority scale between all the considered elements.

- **The logical consistency principle**: The comparisons evoked in the previous paragraph must respect one constraint, namely transitivity. For instance, considering three events A, B and C, if A is better than B and B better than C, then A must be better than C. Moreover, if A is twice better than B and B is three times better than C, then A must be six times better than C: this would constitute a perfectly consistent judgment. Nevertheless, perfect consistency cannot be expected because of the subjective character of the evaluated comparisons and of the changing circumstances: for instance, the same decision-maker might express different choices at two different moments.

The AHP technique thus involves quantitative and qualitative aspects into a unique analysis structure in order to convert the natural thoughts of any human being into an explicit process. This latter is implemented in a decision-support tool that provides objective and reliable results, even under different scenarios occurrence. It is worth noting that, being
Table 1
Value scale for alternative decision comparisons (Saaty, 1980)

<table>
<thead>
<tr>
<th>Comparison intensity</th>
<th>Definition</th>
<th>Explanation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Equally important</td>
<td>Two decisions equally influence the upper-level objective</td>
</tr>
<tr>
<td>3</td>
<td>Moderately more important</td>
<td>One decision is moderately more favorable for the upper-level objective fulfillment</td>
</tr>
<tr>
<td>5</td>
<td>Strongly more important</td>
<td>One decision is strongly more favorable for the upper-level objective fulfillment</td>
</tr>
<tr>
<td>7</td>
<td>Very strongly more important</td>
<td>One decision is significantly more favorable for the upper-level objective fulfillment</td>
</tr>
<tr>
<td>9</td>
<td>Extremely more important</td>
<td>The difference between influences of the two decisions is extremely significant</td>
</tr>
<tr>
<td>2, 4, 6, 8</td>
<td>Intermediate judgment values</td>
<td>When a compromise is necessary to give an intermediary judgment between the previous values</td>
</tr>
</tbody>
</table>

subjective the perceptions of the priority scale provider (i.e. the manager), the AHP method does not integrate the possible existence of an “always true, correct, immutable” decision.

The AHP main steps include [29]:

(1) **Hierarchy design step:** All the elements interfering into the decision-making problem must be determined and structured into levels as a family tree. The first level consists of the primary or main objective while the following ones are devoted to the secondary aims, etc. In the lowest level are the alternatives, i.e. the possible solutions of the multicriteria problem (and so, in the case considered in the study, the non-dominated solutions provided by the Pareto sort): this phase allows clarifying the problem components and their interaction.

(2) **Development of judgment matrices:** One of the main features of the AHP technique is its pairwise comparison working mode, for all the criteria (or alternatives) belonging to the same hierarchical level. Judgment matrices can then be defined from these reciprocal comparisons. The pairwise comparisons are based on a standardized evaluation schemes (cf. next subsection).

(3) **Computing of local priorities:** Several methods for deriving local priorities (i.e. the local weights of criteria or the local scores of alternatives) from judgment matrices have been developed, such as the eigenvector method (EVM), the logarithmic least squares method (LLSM), the weighted least squares method (WLSM), the goal programming method (GPM), etc. Consistency check should be implemented for each judgment matrix.

(4) **Alternative ranking:** An aggregation procedure accounting for all local priorities (thanks to a simple weighted sum) then enables to obtain global priorities regarding the main objective, including global weight of each criteria or global scores of each alternative. The final ranking of the alternatives is determined on the basis of these global priorities.

3.2. Computational details

Assume that \( n \) decision factors are considered in the quantification process of the relative importance of each factor with respect to all the other ones. This problem can be set up as a hierarchy as explained in the previous section. The pairwise comparisons will then be made between each pair of factors at a given level of the hierarchy, regarding their contribution toward the factor at the immediately above level. The comparisons are made on a scale of 1–9, as shown in Table 1. This scale is chosen to support comparisons within a limited range but with sufficient sensitivity (a psychological limit for the human beings to establish quantitative distinction between two elements was proved by psychometric studies). These pairwise comparisons yield a reciprocal \((n, n)\)-matrix \( A \), where \( a_{ii} = 1 \) (diagonal elements) and \( a_{ji} = 1/a_{ij} \).

Suppose that only the first column of matrix \( A \) is provided to state the relative importance of factors 2, 3, \ldots, \( n \) with respect to factor 1. If the judgments were completely consistent, then the remaining columns in the matrix would be completely determined due to the transitivity of the relative importance of the factors. However, there is no consistency except for that obtained by setting \( a_{ji} = 1/a_{ij} \). Therefore, the comparison needs to be repeated for each column of the
matrix, i.e. independent judgments must be made over each pair. Suppose that after all the comparisons are made, the matrix $A$ includes only exact relative weights.

Multiplying the matrix by the vector of weights $w = (w_1, w_2, \ldots, w_n)$ yields:

$$A \cdot w = \begin{bmatrix}
    a_{11} & a_{12} & \cdots & a_{1n} \\
    a_{21} & a_{22} & \cdots & a_{2n} \\
    \vdots & \vdots & \ddots & \vdots \\
    a_{n1} & a_{n2} & \cdots & a_{nn}
\end{bmatrix}
\begin{bmatrix}
    w_1 \\
    w_2 \\
    \vdots \\
    w_n
\end{bmatrix}
= \begin{bmatrix}
    w_1/w_1 & w_1/w_2 & \cdots & w_1/w_n \\
    w_2/w_1 & w_2/w_2 & \cdots & w_2/w_1 \\
    \vdots & \vdots & \ddots & \vdots \\
    w_n/w_1 & w_n/w_2 & \cdots & w_n/w_n
\end{bmatrix}
\begin{bmatrix}
    w_1 \\
    w_2 \\
    \vdots \\
    w_n
\end{bmatrix}
= n \begin{bmatrix}
    w_1 \\
    w_2 \\
    \vdots \\
    w_n
\end{bmatrix}. \quad (5)

Therefore, to recover the overall scale from the matrix of ratios, the EVM was adopted [30]. According to the previous equation, the problem can formulate as $A \cdot w = n \cdot w$ or $(A - nI) = 0$, which represents a system of homogenous linear equations ($I$ is the identity matrix). This system has a nontrivial solution if and only if the determinant of $(A - nI)$ vanishes, meaning that $n$ is an eigenvalue of $A$. Obviously, $A$ has unit rank since every row is a constant multiple of the first row and thus all eigenvalues except one are equal to zero. The sum of the eigenvalues of a matrix equals its trace and in this case, the trace of $A$ equals $n$. So, $n$ is an eigenvalue of $A$ and a nontrivial solution. Usually, the normalized vector is obtained by dividing all the entries $w_i$ by their sum.

Thus, the scale can be recovered from the comparison matrix. In this exact case, the solution is any normalized column of $A$. Notably, matrix $A$ in this case is consistent, indicating that its entries satisfy the condition $a_{jk} = a_{ji}/a_{ki}$ (transitivity property).

3.3. Consistency

However, in actual cases, precise values of $w_i/w_j$ are not available, but their estimates, which in general differ from the ratios of the actual weights, are provided by the decision-maker. The matrix theory illustrates that a small perturbation of the coefficients implies a small perturbation of the eigenvalues. Therefore, an eigenvalue close to $n$, which is the largest eigenvalue $\lambda_{\max}$, should be found since the trace of the matrix (equal to $n$) remains equal to the sum of the eigenvalues while small errors of judgment are made and other eigenvalues are non-zero.

The solution to the problem of the largest eigenvalue, which is the weight eigenvector $w$ that corresponds to $\lambda_{\max}$, when normalized, gives a unique estimate of the underlying ratio scale between the elements of the studied case. Furthermore, the matrix whose entries are $w_i/w_j$ remains a consistent estimate of the “actual” matrix $A$ which may not be consistent. In fact, $A$ is consistent if and only if $\lambda_{\max} = n$. However, the inequality $\lambda_{\max} > n$ always exists. Therefore, the average of the remaining eigenvalues can be used as a “consistency index” (CI) which is the difference between $\lambda_{\max}$ and $n$ divided by the normalizing factor $(n - 1)$.

$$CI = \frac{\lambda_{\max} - n}{n - 1}. \quad (6)$$

The CI of the studied problem is compared with the average RI obtained from associated random matrices of order $n$ to measure the error due to inconsistency [27]. As a rule of thumb, a consistency ratio (CR $= CI/RI$) value of 10% or less is considered as acceptable, otherwise the pairwise comparisons should be revised.

Being concluded the description of the AHP technique, the following section deals with its application to the considered multi-objective problem of batch plant design under demand imprecision.

4. Preliminary design of the analysis

As mentioned in the AHP outline section, the steps of the method implementation are (i) the determination of the main and secondary objectives, in order to define the hierarchy; (ii) the incorporation of the decision-maker’s preferences, in order to finally (iii) generate the priority tables that will support the computations finally providing a ranking of the alternatives.

4.1. Definition of the hierarchy model

Firstly, the primary and secondary objectives have to be chosen to clarify the decision-making problem posture. Obviously, the main goal is finding, among the optimal plant configurations emerging from the Pareto sorting procedure,
the one that best satisfies the decision-maker priorities. Therefore, the global approach has to account for the three criteria considered by the stochastic optimization method, which may be viewed as secondary objectives:

- maximization of the NPV;
- minimization of the production time delay/advance (otherwise formulated such as the maximization of the surface common to both trapezoidal production time $\sum H_i$ and rectangular time horizon $H$);
- maximization of the flexibility index.

However, since these three criteria contain a great amount of information, they deserve further analysis.

4.1.1. Net present value

Let us recall that the NPV criterion is actually a trapezoidal fuzzy quantity. An option could be the use of a defuzzifier to get a real quantity that would not need additional analysis. But, as underlined in previous studies [31], this strategy supposes a considerable information loss, due to the possible variability in the position of the core, support, imprecise zones, etc. This statement is all the more true in the case of asymmetrical fuzzy quantity.

So, in the framework of this study, it seemed preferable to take advantage of all that underlying information in order to define those attributes that are part of the imprecision nature of the resulting fuzzy NPV values. Fig. 7 illustrates these elements, which are explained next.

- Maximal precise point (MPP): Being the extreme left-hand side point in the precise zone, it thus represents the maximal ensured revenue.
- Core (C): This is the precise zone width, i.e. the range of values for which the corresponding NPV is guaranteed. Including this element as an objective enables to compare it with the width of any of the two imprecise zones.
- Maximal point ($P_{\text{Max}}$): This characteristic provides an estimate of the maximal value the NPV can reach. Granting one’s preference to this criterion would characterize a risk-taker decision-maker.
- Right imprecise zone (RIZ): It represents the width of the right-hand side imprecise zone. Focusing on this feature would also characterize a risk-taker manager.
- Minimal point ($P_{\text{Min}}$) and left imprecise zone (LIZ): These two features are the reverse part of the above-defined $P_{\text{Max}}$ and RIZ, and would be logically preferred by a risk-averse manager.

4.1.2. Delay/advance criterion

The second criterion refers to the global production time, which has to coincide with horizon time set within the problem data. From this objective one can derive two decision factors or sub-objectives:

- Maximization of the intersection surface of both trapezoidal production time $\sum H_i$ and rectangular horizon $H$;
- Preferential choice of the cases defined in Fig. 3. Therefore, each case represents another alternative decision that must be evaluated regarding the manager’s preference.
4.1.3. Flexibility index

The flexibility index is a major characteristic of any batch plant. On the one hand, in advance cases, it is formulated as the plant capacity to manufacture an additional product quantity when the initial demand is already satisfied; while on the other hand, in delay cases, it can be viewed as the initial product demand that could not be fulfilled in the fixed time horizon. Thus, three cases arise from the above definition, and constitute three additional alternative decisions:

- **The index is lower than 1**: Being insufficient the plant capacity to satisfy the demand, this case is clearly unfavorable.
- **The index is equal to 1**: This constitutes the perfect solution since the plant configuration is optimally tuned to exactly fulfill the product demand.
- **The index is greater than 1**: This case is also a good option since it provides a major flexibility to the plant, which is able to satisfy an additional demand, for instance, through option contracts. This situation may occur in the advance cases defined in Fig. 3 (namely cases 2, 4, 6 and 8). Note, however, the inherent drawback of this solution: the high treatment capacity, which breeds an early production time, is allowed by oversized processing equipments. Those ones will thus involve a high investment cost that can penalize the NPV criterion.

4.1.4. Decision tree

Therefore, from all the above-mentioned objectives, the resulting decision tree that represents the hierarchical relationships between all those elements is shown in Fig. 8. Let us recall that each of the three main branches of the tree represents one of the criteria considered by the GA. So, the illustrated tree is valid only for the tricriteria decision problem. For both bicriteria cases (NPV–delay/advance and NPV–flexibility index), the inappropriate branch should be removed to get the valid hierarchy model.

4.2. Priority setting: manager profiles

This step is devoted to the determination of the relative importance of each criterion toward the other ones that belong to the same hierarchy level. How above explained, this evaluation is carried out through pairwise comparisons, for which the decision-maker has to express his preference by assigning a numerical value respecting the data stated by Saaty [27] (cf. Table 1). By doing this, each decision-maker has to call for his knowledge, experience, current mood, reality perception, behavior toward risk (averse or taker), etc., so that the decisions he will take might be unique.
In other words, for the same problem and in similar circumstances, it is very likely that two managers will express different judgments on the decision alternatives.

However, this does not mean that the taken decisions will always be radically conflicting: two judgments may only differ in the value assigned to each criterion, since the 9-values scale established by Saaty allows nuances in the expression of similar preferences. This remark thus justifies the possibility to design a reduced number of “preference profiles” that globally coincide in the decisions but might slightly differ for the priority value assignment. Besides, more arguments support the creation of preference profiles:

- Considering the number of possible values to assign to each pairwise comparison, when the number of criteria and of alternatives to account for increases, the possible preference number is subjected to an exponential explosion. So, it is few reasonable to try to take all these possibilities into account.

- Among all those preference options, a huge ratio of them is not coherent. For instance, a priority configuration aiming at maximizing the \( \text{NPV} \) criterion but assigning, at the same time, a high preference to the minimal point \( P_{\text{Min}} \) or to the left-hand side imprecise zone (LIZ), would reach contradiction.

- Many of the possible combinations are quite similar, and it is useless to examine all of them. A great number of these cases can therefore be removed or included into a more general profile.

So, it turns out to be more practical to generate a few coherent strategies than considering all the possible options. This is done through the strategy generation table method, developed by Howard [32], which helps to bring out a reduced profile number. The strategy generation table method is used for modeling a decision situation, when several inter-related decisions are identified: then, it turns out to be more practical to generate a few coherent strategies taking an alternative of every individual decision, than considering all the possible combinations of alternatives. The advantage of the strategy generation table method is the form of complementary modeling that allows generating strategies or preference profiles. In our case, the profiles are proposed by the individual preferences of the process designer, taking into account the three decision criteria and without considering a sample size of the profiles.

An example is provided in Fig. 9 for the first bicriteria case, i.e. \( \text{NPV} \)-delay/advance criteria. The remaining strategy tables and resulting profile lists (for the second bicriteria case and for the tricriteria case) are available in Appendix B.

Note that profile 1 represents a completely neutral decision posture, the preferences for all criteria being equal. Besides, even though it does not appear in any of the strategy tables, a “moderate superiority” importance is in all cases assigned to the maximal precise point (MPP), toward the other objectives of the same level.
Table 2
Strategy for the NPV–delay/advance optimization expressed verbally

<table>
<thead>
<tr>
<th>Configuration</th>
<th>NPV</th>
<th>Delay/advance</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>Equal</td>
<td>Strong</td>
</tr>
<tr>
<td>Delay/advance</td>
<td>–</td>
<td>Equal</td>
</tr>
</tbody>
</table>

Table 3
Strategy for the NPV–delay/advance optimization expressed numerically

<table>
<thead>
<tr>
<th>Configuration</th>
<th>NPV</th>
<th>Delay/advance</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>NPV</td>
<td>1</td>
<td>5</td>
<td>0.83333</td>
</tr>
<tr>
<td>Delay/advance</td>
<td>1/2</td>
<td>1</td>
<td>0.16667</td>
</tr>
</tbody>
</table>

In the above-illustrated bicriteria study, seven profiles are highlighted. The same is true for the other bicriteria case, but when tackling the tricriteria one, the profile number increases to 13. This trend, due to the complexity involved by the higher number of considered criteria, underlines the importance of using the generation strategy tables to bind the number of scenarios.

4.3. Software implementation

Since the various objectives were defined and organized in a hierarchy, the study bases are set to begin the computations. However, as a support for the previously developed methodology and its mathematical implications, this study also proposes a software tool. This one enables to handle the comparisons of comparison matrices and of the resulting eigenvectors within a more simple and friendly environment. Furthermore, the software automatically checks out the consistency of the matrices defined by the user, and sends back an error message if necessary. The completed software can be thus seen as an efficient decision-support tool for managers, using both Pareto-optimal solutions and decision-maker expertise to provide the best solution to the multicriteria problem.

For the sake of illustration, an example, referring to the bicriteria optimization \{NPV–delay/advance criterion\}, is detailed here. Profile 2 will stand as a basis for this illustration. The first step is to define the relative importance of the two objectives “NPV” and “delay/advance”. Table 2 gives an example of a possible strategy (remember that profile 2 gives a greater weight to the NPV) expressed verbally, while Table 3 provides the corresponding values deduced from the table stated by Saaty (cf. Table 1). Moreover, Table 3 also shows the computed eigenvector, allowing the comparison of the two objectives within this level.

For the reader to make himself an idea of the software appearance and conviviality, the window corresponding to this decision level is shown in Fig. 10.

In the same way, the comparison matrices for the remaining decision sublevels are presented next (Tables 4–6). Table 4 shows a quite balanced behavior, between risk-taker and risk-averse, but however giving the preference to the precise zones of the NPV: the Maximal precise point and the Core of the fuzzy number.

Table 5 highlights the very strong priority granted to common surface, thus tending to prefer just-in-time solutions. This trend is confirmed in Table 6, where case 1 is very strongly preferred to all the other ones.

Fig. 11 finally shows the resulting hierarchy tree with the associated eigenvectors, which represent the weights of each objective and sub-objective in the final decision.

5. Computational results

This section deals with the results obtained with the above-developed methodology. Let us recall that, apart from the decision matrices defined by the decision-maker, the basic information on which lies the study is constituted by the non-dominated solutions brought out by the GA.
Fig. 10. Software window—preference assignment for objectives NPV and delay/advance.

Table 4
Comparison matrix and eigenvector for the NPV sublevel

<table>
<thead>
<tr>
<th></th>
<th>MPP</th>
<th>C</th>
<th>$P_{\text{Max}}$</th>
<th>RIZ</th>
<th>$P_{\text{Min}}$</th>
<th>LIZ</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>MPP</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>3</td>
<td>0.2653</td>
</tr>
<tr>
<td>C</td>
<td>1</td>
<td>1</td>
<td>5</td>
<td>5</td>
<td>7</td>
<td>7</td>
<td>0.4040</td>
</tr>
<tr>
<td>$P_{\text{Max}}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0.1132</td>
</tr>
<tr>
<td>RIZ</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>3</td>
<td>3</td>
<td>0.1132</td>
</tr>
<tr>
<td>$P_{\text{Min}}$</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.522</td>
</tr>
<tr>
<td>LIZ</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0522</td>
</tr>
</tbody>
</table>

Table 5
Comparison matrix and eigenvector for the delay/advance sublevel

<table>
<thead>
<tr>
<th></th>
<th>Surface</th>
<th>Cases</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface</td>
<td>1</td>
<td>7</td>
<td>0.8750</td>
</tr>
<tr>
<td>Cases</td>
<td>1/7</td>
<td>1</td>
<td>0.1250</td>
</tr>
</tbody>
</table>

Table 6
Comparison matrix and eigenvector for the cases sublevel

<table>
<thead>
<tr>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
<th>7</th>
<th>8</th>
<th>Eigenvector</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>7</td>
<td>0.5000</td>
</tr>
<tr>
<td>2</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0714</td>
</tr>
<tr>
<td>3</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0714</td>
</tr>
<tr>
<td>4</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0714</td>
</tr>
<tr>
<td>5</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0714</td>
</tr>
<tr>
<td>6</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0714</td>
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<tr>
<td>7</td>
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<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0714</td>
</tr>
<tr>
<td>8</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>0.0714</td>
</tr>
</tbody>
</table>
However, the asymmetrical shape of the uncertain demand shape must be underlined, since these data will have implications on the final result. This choice is justified by the fact that, in real industrial cases, a symmetrical demand is highly unlikely to occur.

This section tackles the bicriteria NPV–delay/advance problem, firstly focusing on the previously detailed profile 2 and, in a second step, providing global results. Then, a discussion is proposed for the other bicriteria case, and finally for the tricriteria computations.

5.1. Bicriteria computations: NPV–delay/advance

5.1.1. Focusing on profile 2

For this case, the number of non-dominated solutions resulting from the Pareto sort is equal to 238. Obviously, detailed results are not given for all these alternatives but only for a few of them. These constitute a reduced sample that enables to provide explicit details on the realized calculations. The considered alternative set is shown in Table 7, with the various criteria that are evaluated through the AHP method.

Actually, it should seem quite strange that no case 1 were got from the optimization, since profile 2 grants priority to just-in-time configurations. This trend is due to the initial demand parameter: the asymmetrical shape results, after the model computations available in Appendix A, in NPV trapezoids that cannot be completely included in the time horizon rectangle (the NPV support width is greater than the H one). Further computations with symmetrical demands are available in Appendix C, and however proved that in this study configuration, case 1 solutions can be obtained.

---

Table 7
Set of tested alternatives provided by the GA (NPV–delay/advance)

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>NPV</th>
<th>Comm. surface</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>736 120.6</td>
<td>817 367.2</td>
<td>906 144.4</td>
</tr>
<tr>
<td>2</td>
<td>736 098.6</td>
<td>817 337.5</td>
<td>906 103.9</td>
</tr>
<tr>
<td>3</td>
<td>736 178.3</td>
<td>817 417.1</td>
<td>906 183.5</td>
</tr>
<tr>
<td>4</td>
<td>735 126.6</td>
<td>816 344.1</td>
<td>905 082.6</td>
</tr>
<tr>
<td>5</td>
<td>733 953.8</td>
<td>815 213.6</td>
<td>904 008.2</td>
</tr>
</tbody>
</table>
Firstly considering the \( NPV \) criterion, the AHP technique breaks it down into various essential elements related to the shape of the considered fuzzy quantities. All these features are given in Table 8. However, it is to note that these GA solutions cannot be used in their rough state, but need a scaling treatment, since the data do not all have the same range of magnitude. A normalization procedure is thus carried out on each basic criterion, consisting in computing the sum of all the quantities related to one factor, and then dividing each element by this sum. The multiplication of the arising matrix, \( NORM_{NPV} \), by the eigenvector computed in the previous section for the \( NPV \) criterion, \( EVECT_{NPV} \), results in the weight vector for the secondary objective \( WV_{NPV} \), as shown in the following equation:

\[
NORM_{NPV} \times EVECT_{NPV} = WV_{NPV} = \begin{bmatrix}
0.20007194 \\
0.20005347 \\
0.200061119 \\
0.200061119 \\
0.199891482
\end{bmatrix}.
\] (7)

Then, a similar procedure is implemented for the delay/advance criterion, i.e. common surface and cases. For the common surface, the same normalization method can be applied, but, at a lower level, it cannot be used for the cases normalization: since they express a qualitative judgment, their value was replaced directly by the value obtained from the Saaty scale, i.e. 0.0714 (since all are case 2). The classical normalization procedure is then used, logically resulting, for all alternatives, in a value equal to 1/5 = 0.2.

So, the matrix corresponding to the normalized alternatives according to the delay/advance criterion, \( NORM_{d/a} \), is actually composed of the two vectors representing the normalized common surface and cases. Multiplying it by the delay/advance eigenvector \( EVECT_{d/a} \), the weight vector for the delay/advance \( WV_{d/a} \) criterion is obtained:

\[
NORM_{d/a} \times EVECT_{d/a} = WV_{d/a} = \begin{bmatrix}
0.199953928 \\
0.200034286 \\
0.199921784 \\
0.200039644 \\
0.200050358
\end{bmatrix}.
\] (8)

Finally, combining the weight vectors associated to each criterion into a single matrix and multiplying it by the previously computed global eigenvector, the evaluation of each alternative is obtained, and presented in Table 9.

A ranking can thus be established according to the above-presented evaluations: alternative 1 is the better option for profile 2 decision-makers. A more accurate observation of each alternative reveals that the ranking respects the local priorities granted for \( NPV \) and delay/advance elements (\( C < RIZ < LIZ \) and surface > cases), as well as the global priority (\( NPV > \) delay/advance). The corresponding plant configuration is available in Table 10, but further analysis proves that all five alternatives show the same global structure (number of parallel units per stage), the difference lying in the item sizes.

5.1.2. Remaining profiles and interpretation

Following the same methodology, the best alternative for each profile was determined, and the results are summed up in Table 11. It firstly can be noticed that some profiles derive in the same solution: profiles 1 and 4 coincide, as well as profiles 6 and 7. Profiles 2, 3 and 5 differ from all the other ones.
Table 9
Alternative evaluation for the global objective

<table>
<thead>
<tr>
<th>Alternatives</th>
<th>Evaluation</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.200052267</td>
</tr>
<tr>
<td>2</td>
<td>0.200050272</td>
</tr>
<tr>
<td>3</td>
<td>0.200037892</td>
</tr>
<tr>
<td>4</td>
<td>0.199941602</td>
</tr>
<tr>
<td>5</td>
<td>0.199917966</td>
</tr>
</tbody>
</table>

The boldface values highlight the best alternative.

Table 10
Plant configuration for alternative 1

<table>
<thead>
<tr>
<th>Batch stages</th>
<th>1</th>
<th>2</th>
<th>3</th>
<th>4</th>
<th>5</th>
<th>6</th>
</tr>
</thead>
<tbody>
<tr>
<td>Item size ($V_j$)</td>
<td>8100</td>
<td>9950</td>
<td>9950</td>
<td>5500</td>
<td>7600</td>
<td>10000</td>
</tr>
<tr>
<td>Parallel units ($m_j$)</td>
<td>2</td>
<td>2</td>
<td>2</td>
<td>3</td>
<td>1</td>
<td>2</td>
</tr>
<tr>
<td>Semi-continuous stages</td>
<td>1</td>
<td>2</td>
<td>3</td>
<td>4</td>
<td>5</td>
<td>6</td>
</tr>
<tr>
<td>Item size ($R_k$)</td>
<td>550</td>
<td>3250</td>
<td>1950</td>
<td>7250</td>
<td>6500</td>
<td>450</td>
</tr>
<tr>
<td>Parallel units ($n_k$)</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
<td>1</td>
</tr>
</tbody>
</table>

Table 11
Best alternatives according to preference profiles ($NPV$–delay/advance)

<table>
<thead>
<tr>
<th>Preference profiles</th>
<th>$NPV$</th>
<th>Comm. surface</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile 1</td>
<td>736.0986</td>
<td>817.3375</td>
<td>906.1039</td>
</tr>
<tr>
<td>Profile 2</td>
<td>736.1206</td>
<td>817.3672</td>
<td>906.1444</td>
</tr>
<tr>
<td>Profile 3</td>
<td>736.2377</td>
<td>817.4843</td>
<td>906.2615</td>
</tr>
<tr>
<td>Profile 4</td>
<td>736.0986</td>
<td>817.3375</td>
<td>906.1039</td>
</tr>
<tr>
<td>Profile 5</td>
<td>736.1863</td>
<td>817.4251</td>
<td>906.1915</td>
</tr>
<tr>
<td>Profile 6</td>
<td>733.3621</td>
<td>814.5873</td>
<td>903.3367</td>
</tr>
<tr>
<td>Profile 7</td>
<td>733.3621</td>
<td>814.5873</td>
<td>903.3367</td>
</tr>
</tbody>
</table>

But more roughly, three groups of similar profiles clearly stand out. On the one hand, profiles 6 and 7 show lower $NPV$ values with respect to the other profiles (about 0.4%). However, as predictable by observing the associated profile priorities, this has compensation in terms of production time, since the products can be delivered in advance (case 2) with respect to the horizon time, and the common surface is the highest toward all other profiles.

On the other hand, profiles 3 and 5 present a good likeness, both proposing the highest $NPV$ values. The price of this good result is significant regarding the production time, since these profiles are characterized by the case 3 (delay); moreover, the common surface is low meaning that the lateness is quite important.

Finally, profiles 1, 2 and 4 seem to constitute the best agreement between the above-described extreme cases: the production time is early, with a common surface almost equal to that of profiles 6 and 7. Besides, the $NPV$ values are similar to those of profiles 3 and 5. Thus interpreting these results in terms of risk, profiles 6 and 7 would characterize a risk-averse manager, profiles 3 and 5 a risk-taker manager, while profiles 1, 2 and 4 strive for intermediate, compromise solutions.

5.2. Bicriteria computations: $NPV$–flexibility

Table 12 gives the best result for the seven profiles defined for the bicriteria ($NPV$–flexibility). (Note, however, that these seven profiles have got nothing to do with those of the previously treated bicriteria case.) In this case, profiles 2, 3 and 4 on the one hand, and profiles 5 and 7 on the other hand, present identical solutions. Profiles 1 and 6 show very
Table 12
Best alternatives according to preference profiles (*NPV*–flexibility)

<table>
<thead>
<tr>
<th>Preference profiles</th>
<th><em>NPV</em></th>
<th>Flexibility index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile 1</td>
<td>532,922.7</td>
<td>614,261.0</td>
</tr>
<tr>
<td>Profile 2</td>
<td>712,518.7</td>
<td>882,677.8</td>
</tr>
<tr>
<td>Profile 3</td>
<td>712,518.7</td>
<td>882,677.8</td>
</tr>
<tr>
<td>Profile 4</td>
<td>712,518.7</td>
<td>882,677.8</td>
</tr>
<tr>
<td>Profile 5</td>
<td>733,228.5</td>
<td>903,419.7</td>
</tr>
<tr>
<td>Profile 6</td>
<td>475,996.9</td>
<td>646,639.9</td>
</tr>
<tr>
<td>Profile 7</td>
<td>733,228.5</td>
<td>903,419.7</td>
</tr>
</tbody>
</table>

Table 13
Best alternatives according to preference profiles (tricriteria analysis)

<table>
<thead>
<tr>
<th>Preference profiles</th>
<th><em>NPV</em></th>
<th>Comm. surface</th>
<th>Case</th>
<th>Flexibility index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile 1</td>
<td>700,577.8</td>
<td>871,917.6</td>
<td>643.64</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 2</td>
<td>688,247.8</td>
<td>938,205.9</td>
<td>649.42</td>
<td>1</td>
</tr>
<tr>
<td>Profile 3</td>
<td>700,577.8</td>
<td>871,917.6</td>
<td>643.64</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 4</td>
<td>700,577.8</td>
<td>871,917.6</td>
<td>643.64</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 5</td>
<td>700,577.8</td>
<td>871,917.6</td>
<td>643.64</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 6</td>
<td>700,577.8</td>
<td>937,764.6</td>
<td>650.84</td>
<td>1</td>
</tr>
<tr>
<td>Profile 7</td>
<td>447,400.7</td>
<td>937,764.6</td>
<td>650.84</td>
<td>1</td>
</tr>
<tr>
<td>Profile 8</td>
<td>447,400.7</td>
<td>937,764.6</td>
<td>650.84</td>
<td>1</td>
</tr>
<tr>
<td>Profile 9</td>
<td>700,577.8</td>
<td>948,395.0</td>
<td>643.64</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 10</td>
<td>687,806.5</td>
<td>948,395.0</td>
<td>643.64</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 11</td>
<td>598,703.2</td>
<td>948,395.0</td>
<td>643.64</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 12</td>
<td>586,504.3</td>
<td>948,395.0</td>
<td>643.64</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 13</td>
<td>586,504.3</td>
<td>948,395.0</td>
<td>643.64</td>
<td>1.01</td>
</tr>
</tbody>
</table>

It is possible, such as done in the previous section, to classify the profiles according to their behavior toward risk. Stating that a greater flexibility is synonymous with aversion to risk, and consequently involves lower incomes, the profiles can be ranked in the following way, from the most risk-taker one to the most risk-averse: profiles 5–7, profiles 2–4, profile 1 and profile 6.

Besides, the numerical values highlight that, unlike for the (*NPV*–delay/advance) computations, there exists great gaps between the four different configurations proposed by the AHP methodology: the range between extreme *NPV* values is [476.10⁶; 733.10⁶], while the flexibility index varies between 1 and 1.48. This trend naturally leads managers to more drastic decisions and, therefore, to more severe implications.

5.3. Tricriteria computations

For the analysis of tricriteria computations, Table 13 gives the best alternatives obtained for each of the 13 profiles and like in the above-treated bicriteria (*NPV*–flexibility) case, no solution is characterized by delayed production times, because no profile grants priority to flexibility indexes lower than 1 (see Appendix B). Thus, this point emphasizes the antagonism between the delay/advance criterion and the flexibility one: the former is pushing the solutions toward the just-in-time configuration (by maximizing the common surface), while the latter is pulling them toward earliest completion times, in order to obtain a greater flexibility.

This explains why, for instance, the solution corresponding to profile 11 provides a better flexibility and greater incomes than profiles 12 and 13. Therefore, it is more difficult in this tricriteria case to determine a ranking of the profiles based on behavior toward risk.
However, general comments can be underlined. Firstly, profiles 1, 3, 4, 5, 6, 9 on the one hand, profiles 7 and 10 on the other hand, and finally profiles 12 and 13, show identical solutions. Moreover, it can be considered that profile 2 is very close to profiles 7 and 10, and so included in the same category.

The first group (profiles 1, 3, 4, 5, 6, 9) shows the greatest NPV values, high common surface and a quite low flexibility, that would characterize a risk-taker decision-maker. Profiles 2, 7 and 10 is slightly comparable, but with a higher common surface, and lower flexibility and incomes. Profiles 11 is, reversely, more averse toward risk, since the incomes are lower and the flexibility greater (with a low common surface). Profiles 12 and 13 are very similar to profiles 2, 7, 11 but with lower NPV values. Finally, profile 8 leads to the most risk-averse decisions, since the NPV values are very low and the flexibility greatly exceeds the other profiles’ one: the common surface is then equal to zero, since case 8 corresponds to a great advance for the production time.

6. Conclusions

This study addresses the issue of multicriteria batch plant design under imprecise demands. The optimization carried out in previous studies with an adapted Genetic Algorithm (GA) provided a huge number of non-dominated solutions, which requires an additional treatment. This new step was implemented through the well-known analytic hierarchy process (AHP), in order to account for the subjective judgments of any manager. Besides, the resulting fuzzy results are decomposed into essential elements to take advantage of the whole information they contain.

The proposed framework thus enables the decision-maker to introduce its preferences into a software that interprets the GA solutions and gives the best alternative according to the chosen priorities. To provide a more complete evaluation of the proposed tool without dealing with all the possible priority combinations, some preference profiles were highlighted and the corresponding solutions were computed. The results analysis underlined some typical trends, more particularly with respect to the manager’s behavior towards risk. Therefore, the proposed decision-making tool is a complete software that uses outer information given by a systematic optimization technique, integrates the managers priorities through a friendly interface, automatically carries out the AHP computations and finally provides the most adapted solutions with all its characteristics (plant configuration, costs and incomes, production time, etc.).

Appendix A. Mathematical model

The aim of this appendix is to present the details of the computations implemented to calculate each of the three criteria involved in the paper.

1. Net present value

As stated in Section 2.2, the NPV is computed according to Eq. (1), recalled here as Eq. (A.1):

$$\text{Max } NPV = -Inv - f + \sum_{p=1}^{n} \frac{(V_p - D_p - A_p)(1 - a) + A_p}{(1 + i)^n} + \frac{f}{(1 + i)^n},$$

(A.1)

Detailing the terms of this equation, the parameter are the taxation rate ($a$), the money actualization rate ($i$) and the time period on which the profit values are considered ($n$), i.e. the plant lifetime. The remaining terms are explained next.

(a) The investment cost $Inv$ is formulated in terms of the optimization variables, which represent the plant configuration:

$$Inv = \sum_{j=1}^{J} (m_j a_j V_j^{x_j}) + \sum_{k=1}^{K} (n_k b_k R_k^{\beta_k}) + \sum_{s=1}^{S} (c_s V_s^{\gamma_s}),$$

(A.2)

where $I, J, K$ are the number of batch stages, semi-continuous stages and storage tanks respectively; and $a_j, b_k, c_s, x_j, \beta_k, \gamma_s$ are cost coefficients.
(b) Incomes coming from the $I$ products:

$$V_p = \sum_{i=1}^{I} C_{Pi} Q_i,$$

where $C_{Pi}$ is the selling cost for each product unit.

(c) Operating cost for the plant production:

$$D_p = \sum_{i=1}^{I} \left[ \left( \sum_{j=1}^{J} C_{Ej} Q_i \right) + C_{Oi} Q_i \right],$$

where $C_{Ej}$ is the operating cost for batch stage $j$, independently from the bath size $B_{is}$, and $C_{Oi}$ is the operating cost from the manufactured products.

(d) The amortization strategy is linear:

$$A_p = \frac{Inv}{n}.$$  \hfill (A.5)

(e) Turnover funds:

$$f = 1.15 Inv.$$ \hfill (A.6)

2. Delay/advance criterion

The second criterion overall appeals to the computation of the production time. This one is derived from the global productivity of the process, which is obtained from the following equations:

(a) Batch size of product $i$ in sub-process $s$:

$$\forall i \in \{1, \ldots, I\}; \forall s \in \{1, \ldots, S\}, \quad B_{is} = \min_{j \in J_s} \left[ \frac{V_j}{S_{ij}} \right].$$ \hfill (A.7)

(b) Operating time for product $i$ in semi-continuous stage $k$:

$$\forall i \in \{1, \ldots, I\}; \forall k \in \{1, \ldots, K_s\}; \forall s \in \{1, \ldots, S\}, \quad \theta_{ik} = \frac{B_{is} D_{ik}}{R_{kn_k}}.$$ \hfill (A.8)

(c) Processing time of product $i$ in batch stage $j$:

$$\forall i \in \{1, \ldots, I\}; \forall j \in \{1, \ldots, J_s\}; \forall s \in \{1, \ldots, S\}, \quad p_{ij} = p^0_{ij} + g_{ij} B_{dj}. $$ \hfill (A.9)

(d) Cycle time for product $i$ in batch stage $j$:

$$\forall i \in \{1, \ldots, I\}; \forall j \in \{1, \ldots, J_s\}, \quad T_{ij} = \frac{\theta_{ik} + \theta_{i(k+1)} + p_{ij}}{m_j},$$ \hfill (A.10)

where $k$ and $k+1$ represent the semi-continuous stages before and after batch stage $j$.

(e) Limiting cycle time for product $i$ in sub-process $s$:

$$\forall i \in \{1, \ldots, I\}; \forall s \in \{1, \ldots, S\}, \quad T_{is}^L = \max_{j \in J_s} [T_{ij}, \theta_{ik}],$$ \hfill (A.11)

where $J_s$ and $K_s$ are respectively the sets of batch and semi-continuous stages in sub-process $s$.

(f) Local productivities for product $i$ in sub-process $s$:

$$\forall i \in \{1, \ldots, I\}; \forall s \in \{1, \ldots, S\}, \quad Prod_{loc_{is}} = \frac{B_{is}}{T_{is}^L}. $$ \hfill (A.12)

(g) The global productivity for each product is deduced from the local productivities computed in each sub-process:

$$\forall i \in \{1, \ldots, I\}, \quad Prod_i = \min_{s \in S} [Prod_{loc_{is}}].$$ \hfill (A.13)
Finally, the production time $H_i$ is calculated from the productivities and demand for each product $i$. Since the latter is a fuzzy quantity, so is $H_i$.

$$\forall i \in \{1, \ldots, I\}, \quad H_i = \frac{Q_i}{\text{Prod}_i}. \quad (A.14)$$

Thus, the total production time is obtained by computing the sum of all $H_i$, and the delay/advance criterion consists in the maximization of the surface common to both trapezoidal $\sum H_i$ and rectangular $H$. According to the cases defined in Fig. 3, the common surface is calculated through the following expressions:

$$\text{Max(Delay crit.)} = \text{common surface} \times \frac{1}{\omega}, \quad (A.15)$$

$$\text{Max(Advance crit.)} = \text{common surface} \times \omega, \quad (A.16)$$

where $\omega$ is a penalization factor.

Besides, the size of intermediate storage tanks, needed for the investment cost computation, is estimated as the highest size difference between the batches treated in two successive sub-processes:

$$\forall s \in \{1, \ldots, S - 1\}, \quad V^* = \max_{i \in I} \left[ S_i \text{Prod}_i (T^L_{is} + T^L_{i(s+1)} - \theta_{ik} - \theta_{i(k+1)}) \right]. \quad (A.17)$$

### 3. Flexibility index

The computation of the flexibility index is detailed here for the advance case, assuming that it is similar in the delay case. The main point of the calculations is that the early production time lets a free amount of time to manufacture some additional product quantity. Fig. A1 illustrates this additional time $\Delta t$. It was assumed that $\Delta t$ is used for an equitable production of all the products initially synthesized by the plant. Thus, the allowed production time is, for each product $i$:

$$H^*_i = \frac{\Delta t}{T_i}. \quad (A.18)$$

Since the plant productivity for each product is known ($\text{Prod}_i$, see Eq. (A.13)), it is possible to deduce the additional amount of manufactured products:

$$Q^*_i = H^*_i \times \text{Prod}_i. \quad (A.19)$$

Finally, the flexibility index is formulated as the ratio between the new total production and the initial demand:

$$\text{flexibility index} = \frac{\sum_{i=1}^{I} (Q_i + Q^*_i)}{\sum_{i=1}^{I} Q_i}. \quad (A.20)$$

### Appendix B. Strategy generation tables

The generation strategy tables for the second bicriteria optimization case ($NPV$–flexibility) and the tricriteria optimization case are showed in Figs. B1 and B2.
Fig. B1. Profile strategy table for the bicriteria optimization \(\{NPV\text{ vs. flexibility index}\}.

Fig. B2. Profile strategy for the tricriteria optimization.
Appendix C. Results with symmetrical demands

Best alternatives according to preference profiles for $NPV$-delay/advance, $NPV$-flexibility, and tricriteria analysis are presented in Tables C1–C3.

Table C1
Best alternatives according to preference profiles ($NPV$–delay/advance)

<table>
<thead>
<tr>
<th>Preference profiles</th>
<th>NPV</th>
<th>Comm. surface</th>
<th>Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile 1</td>
<td>784 834.7</td>
<td>815 665</td>
<td>877 325.8</td>
</tr>
<tr>
<td>Profile 2</td>
<td>784 834.7</td>
<td>815 665</td>
<td>877 325.8</td>
</tr>
<tr>
<td>Profile 3</td>
<td>784 819.7</td>
<td>815 657.9</td>
<td>877 334.2</td>
</tr>
<tr>
<td>Profile 4</td>
<td>784 834.7</td>
<td>815 665</td>
<td>877 325.8</td>
</tr>
<tr>
<td>Profile 5</td>
<td>784 819.7</td>
<td>815 657.9</td>
<td>877 334.2</td>
</tr>
<tr>
<td>Profile 6</td>
<td>784 834.7</td>
<td>815 665</td>
<td>877 325.8</td>
</tr>
<tr>
<td>Profile 7</td>
<td>784 834.7</td>
<td>815 665</td>
<td>877 325.8</td>
</tr>
</tbody>
</table>

Table C2
Best alternatives according to preference profiles ($NPV$–flexibility)

<table>
<thead>
<tr>
<th>Preference profiles</th>
<th>NPV</th>
<th>Flexibility index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile 1</td>
<td>534 890.0</td>
<td>658 781.6</td>
</tr>
<tr>
<td>Profile 2</td>
<td>824 227.1</td>
<td>947 267.2</td>
</tr>
<tr>
<td>Profile 3</td>
<td>776 597.7</td>
<td>899 430.9</td>
</tr>
<tr>
<td>Profile 4</td>
<td>824 227.1</td>
<td>947 267.2</td>
</tr>
<tr>
<td>Profile 5</td>
<td>824 227.1</td>
<td>947 267.2</td>
</tr>
<tr>
<td>Profile 6</td>
<td>534 890.0</td>
<td>658 781.6</td>
</tr>
<tr>
<td>Profile 7</td>
<td>824 227.1</td>
<td>947 267.2</td>
</tr>
</tbody>
</table>

Table C3
Best alternatives according to preference profiles (tricriteria analysis)

<table>
<thead>
<tr>
<th>Preference profiles</th>
<th>NPV</th>
<th>Comm. surface</th>
<th>Case</th>
<th>Flexibility index</th>
</tr>
</thead>
<tbody>
<tr>
<td>Profile 1</td>
<td>779 918.3</td>
<td>803 694.3</td>
<td>100</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 2</td>
<td>754 837.5</td>
<td>879 276.5</td>
<td>343.32</td>
<td>2</td>
</tr>
<tr>
<td>Profile 3</td>
<td>754 837.5</td>
<td>879 276.5</td>
<td>343.32</td>
<td>2</td>
</tr>
<tr>
<td>Profile 4</td>
<td>779 918.3</td>
<td>803 694.3</td>
<td>100</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 5</td>
<td>779 918.3</td>
<td>803 694.3</td>
<td>100</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 6</td>
<td>779 918.3</td>
<td>803 694.3</td>
<td>100</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 7</td>
<td>744 506.8</td>
<td>869 710.7</td>
<td>352.27</td>
<td>2</td>
</tr>
<tr>
<td>Profile 8</td>
<td>696 990</td>
<td>822 301.7</td>
<td>1000</td>
<td>1</td>
</tr>
<tr>
<td>Profile 9</td>
<td>779 918.3</td>
<td>803 694.3</td>
<td>100</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 10</td>
<td>779 918.3</td>
<td>803 694.3</td>
<td>100</td>
<td>1.01</td>
</tr>
<tr>
<td>Profile 11</td>
<td>696 990</td>
<td>822 301.7</td>
<td>1000</td>
<td>1</td>
</tr>
<tr>
<td>Profile 12</td>
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</tr>
<tr>
<td>Profile 13</td>
<td>670 687.3</td>
<td>798 663.2</td>
<td>1000</td>
<td>1</td>
</tr>
</tbody>
</table>

References


